Technical Comments

Comment on "Drag Reduction Factor Due to Ground Effect"

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In a recent Note, Suh and Ostowari[1] presented an expression for the reduction of induced drag when a wing approaches the ground. Their e coefficient in Eq. (3) cannot be called the Oswald wing efficiency factor since this is a factor, only slightly less then unity, that increases C_{D_i} in order to approximate the effects of flow interference and separation. Instead, their e factor may be more properly considered as an adjustment for the assumed circulation constant k_o. For example, Houghton (author's Ref. 10, p. 286 in 1984, 3rd ed., or p. 373 in 1970, 2nd ed.) assumes that k_o has exactly one-half the strength assumed by the authors[1]; consequently, $e = 1/4$ gives exactly Houghton's results. In all cases, however, Eq. (3) cannot give a good approximation of ϕ for small values of h/s since it becomes negative, which of course is physically impossible for a finite value of h/s. For example, if $h/s < 0.067$ then $\phi < 0$ for $e = 1$. The failure of Eq. (3) to give a good approximation for real ground effects can be shown by comparing it with the numerical values obtained by integrating the elliptic load distributions corresponding trailing vortex system, as in Ref. 2. For $h/s = 0.2$ we have $\phi = (1 - \sigma) = 0.516$, which can be matched by Eq. (3) only for $e = 0.853$. Then for $h/s = 0.05$ we now have $\phi = (1 - \sigma) = 0.220$, which corresponds to $e = 0.689$; consequently, Eq. (3) is not a very satisfactory approximation for ground effect. The ϕ (Prandtl) given by the authors is Prandtl's numerical approximation for the values he obtained by graphical integration. The more accurate numerical values given in Ref. 2 were obtained by the numerical integration process described in Ref. 3, and may be very closely approximated:

$$\phi \approx [1 + 0.187s/h]^{-1}; \text{ for } (h/s) < 1/2 \tag{1}$$

The approximations being developed apply primarily to unswept wings having a relatively large aspect ratio ($\text{Æ} > 2$).

When $(h/s) > 1$ the horseshoe vortex system provides a better approximation. The ϕ (McCormick) given by the authors[1] repeats the obvious misprint in Eq. (7.2) on page 420 of McCormick (author's Ref. 9), where the term $(8h/s)^2 = (16h/b)^2$ should be replaced by $(8h/\pi s)^2$. This corrected equation gives $\phi = 0.206$ for $s/h = 5$ (or $h/b = 0.1$), agreeing with Fig. 7.4 on page 422 of McCormick. This approximation gives too large a ground effect because it is based on $\Delta w_i(o)/w_i(o)$, with $w_i(o) = k_o/2\pi s'$. A much better approximation is obtained by using $\Delta w_i(o)/\bar{w}_i$, where $\bar{w}_i = U\alpha_i = UC_L/\pi \text{Æ}$, and $k_o = L/\rho U(2s)$ corresponds to the total circulation strength that was required to produce the elliptic load distribution over the original span $(2s)$. Then assuming the horseshoe vortex has the smaller span $(2s')$ we obtain:

$$(1 - \phi) = \sigma = \frac{\Delta w_i(o)/U}{C_L/\pi \text{Æ}} = \frac{2/\pi}{1 + (2h/s')^2} \tag{2}$$

Therefore, with $s' = \pi s/4$, we obtain:

$$\phi = [1 - 2/\pi + (8h/\pi s)^2][1 + (8h/\pi s)^2]^{-1} \tag{3}$$

This approximation gives $\phi = 0.4945$ for $(h/s) = 1/5$, as compared to $\phi = 0.5168$ from Eq. (1), or $(1 - \sigma) = 0.5157$ from Ref. 3. However, for $(h/s) > 1/2$, the preceding Eq. (3) gives better numerical values than any of the other approximations, probably because $\phi(o) = (1 - 2/\pi) > 0$.

Finally, the induced drag coefficient with ground effect can now be approximated:

$$C_{D_i} \approx \phi C_L^2/\pi e_o \text{Æ} \tag{4}$$

where the Oswald efficiency factor ($e_o \approx 1$) can only be evaluated by flight or wind tunnel tests, since its value is also affected by the proximity to the ground.

References

[1]Suh, Y. B. and Ostowari, C., "Drag Reduction Factor Due to Ground Effect," *Journal of Aircraft*, Vol. 25, Nov. 1988, pp. 1071–1072.

[2]Laitone, E. V., "Positive Tail Loads for Minimum Induced Drag of Subsonic Aircraft," *Journal of Aircraft*, Vol. 15, Dec. 1978, pp. 837–842.

[3]Laitone, E. V., "Extension of Prandtl's Biplane Theory to Wing-Tail Combinations," *Canadian Aeronautics and Space Journal*, Vol. 25, No. 3, 1979, pp. 278–285.

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